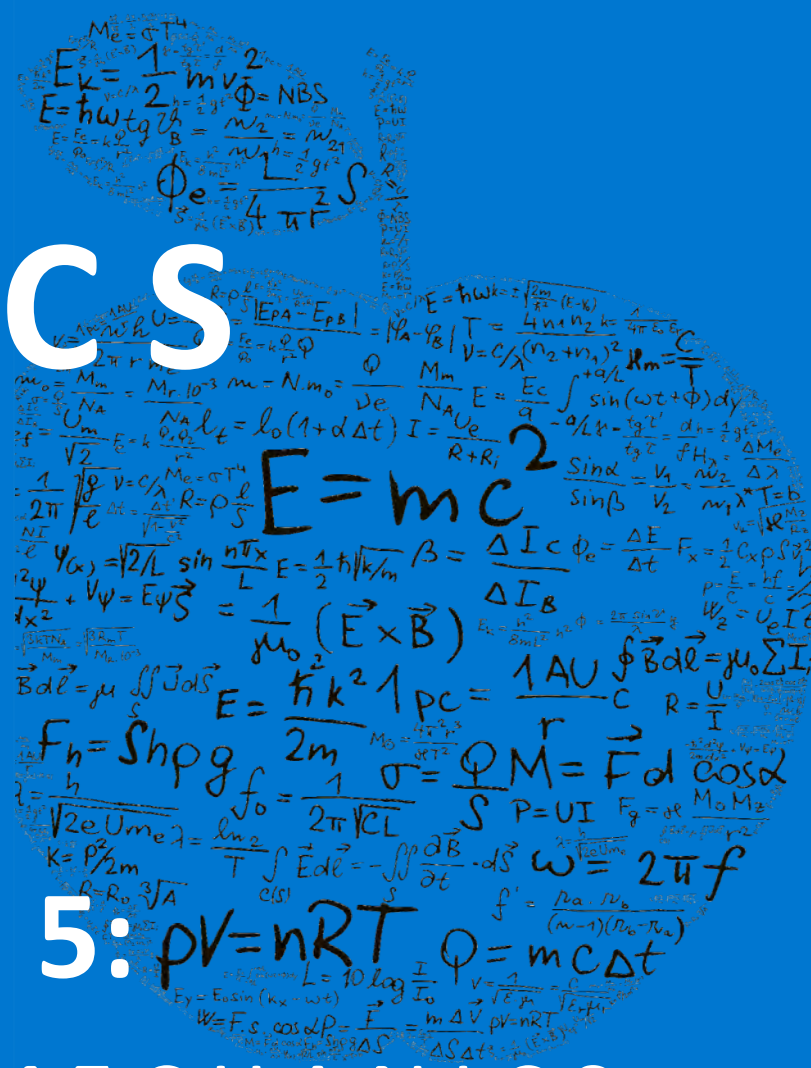


NAME: \_\_\_\_\_

# HSC PHYSICS



# MODULE 5: ADVANCED MECHANICS

## FOCUS 2: PROJECTILE MOTION II

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# Projectile Motion II

**By the end of this booklet, students should be able to:**

- Understand how the equations of uniformly accelerated motion can be applied to Type 2 projectile motion in a variety of scenarios, including:
  - Solving for maximum height and time to maximum height
  - Solving for time of flight and range of the projectile
  - Solving for speed / velocity at a particular point in time or at a particular displacement
  
- Understand how the equations of uniformly accelerated motion can be applied to Type 3 projectile motion in a variety of scenarios, including:
  - Solving for maximum height and time to maximum height
  - Use of quadratic formula to solve for time to maximum height
  - Solving for time of flight and range of the projectile
  - Use of quadratic formula to solve for time of flight
  - Solving for speed / velocity at a particular point in time or at a particular displacement
  - Solving for the above when the landing surface is not flat
  
- Understand how the equations of uniformly accelerated motion can be applied to Type 4 projectile motion in a variety of scenarios, including:
  - Breaking down the question and expressing in terms of variables what the question is asking
  - Identifying conditions which occur synchronously
  - Identifying any assumptions made
  
- Be able to interpret and analyse secondary data, including stroboscopic photos of objects undergoing projectile motion, including solving for:
  - Initial velocity of projectile
  - Final velocity of projectile
  - Stroboscope frequency / period
  - Range of projectile
  - Acceleration due to gravity
  - Relationship(s) between variables in projectile motion equations

# Projectile Motion – Type 2

## Recall Concept I

A Type 2 projectile is a projectile which is launched from a certain point and then strikes another point, such that the change in y-displacement from the beginning to the end of its journey is equal to zero.

## Sigma Sample 5.2.1

Σ

A ball is launched from the ground at  $30\text{ms}^{-1}$  at an angle of  $65^\circ$  above the horizontal. The ball then returns to the ground.

**i. What do you already know?**

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**ii. Determine  $t_{\text{max}}$  OR  $\Delta y_{\text{max}}$**

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**iii. Determine TOF and/or the range of the object**

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**iv. Determine the velocity of the object at impact**

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1. A student kicks a ball from the ground giving it a velocity of  $54 \text{ m s}^{-1}$  at an angle of  $37^\circ$  above the horizontal.

(a) What are the vertical and horizontal components of the ball's initial velocity?  **$32.5 \text{ ms}^{-1} / 43.1 \text{ ms}^{-1}$**

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(b) How long is the ball in the air? **6.6 s**

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(c) What is the horizontal distance covered by the ball while in flight? **284 m**

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(d) What velocity does the ball have at the top of its trajectory?  **$43 \text{ ms}^{-1}$  horizontally**

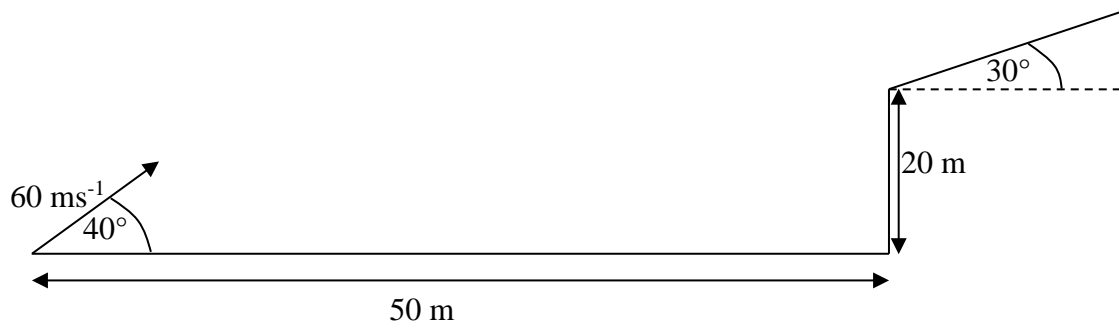
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5. An object is launched from the ground at  $60 \text{ ms}^{-1}$  and lands on a hill which is inclined at  $30^\circ$  as shown below:



Find the following:

- (a) The horizontal distance from the origin to the point where the projectile lands.

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- (b) How far up the hill the ball lands.

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- (c) The speed at which the ball strikes the hill.

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# Projectile Motion – Type 4

Type 4 projectiles are generally the hardest type of projectile motion and often involve determining multiple conditions which must be true in order for the projectile motion to be completed.

It is the determination of the condition(s) which make this type of projectile motion difficult, not the actual mathematics.

Very often your solution will involve you realising that 2 conditions are occurring at the same value of  $t$ .

**Sigma Sample 5.2.3:**  $\Sigma \Sigma \Sigma$

A golfer strikes a ball at  $40 \text{ ms}^{-1}$  at an angle of  $40^\circ$  above the horizontal. A 20m tall tree is 25m away. Does the ball clear the tree?

(a) Firstly, you need to be able to express, in physics language, what this question is actually asking.

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(b) Then solve it ☺

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While playing tennis, Rohan serves a ball with a velocity of  $40 \text{ ms}^{-1}$  at an angle of  $5^\circ$  below the horizontal. The racquet strikes the ball a distance of 2.50m above the ground. The net is 11.9m away and is of height 0.92m. Does the ball clear the net? Justify your answer with relevant equation(s).

(a) Express, in physics language, what this question is actually asking.

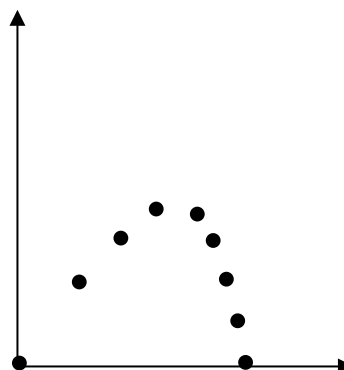
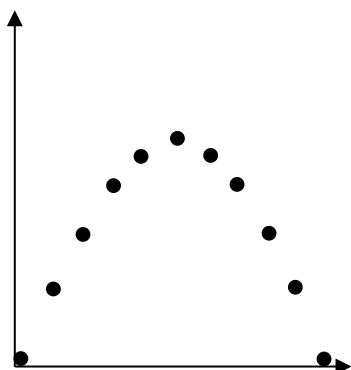
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(b) Then solve it ☺

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5. A practical investigation was performed where the impact of air resistance on projectile motion was being investigated.

A projectile was launched with a speed of  $u$  at an angle of  $45^\circ$  in both an evacuated chamber without air resistance and in the classroom. The following images were obtained by using a stroboscope.



Discuss how the effects of air resistance on the projectiles are demonstrated in the diagrams above.

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# Projectile Motion – Experimental analysis

It is extremely common for a practical task to be conducted on Projectile Motion as there are such a vast array of experiments which you can perform.

Below are some examples of experiments which you should be familiar with for projectile motion.

## Sample Problem 5.2.4 (Practical Investigation)

Students are asked to find a relationship between the angle of elevation of a projectile and its range.

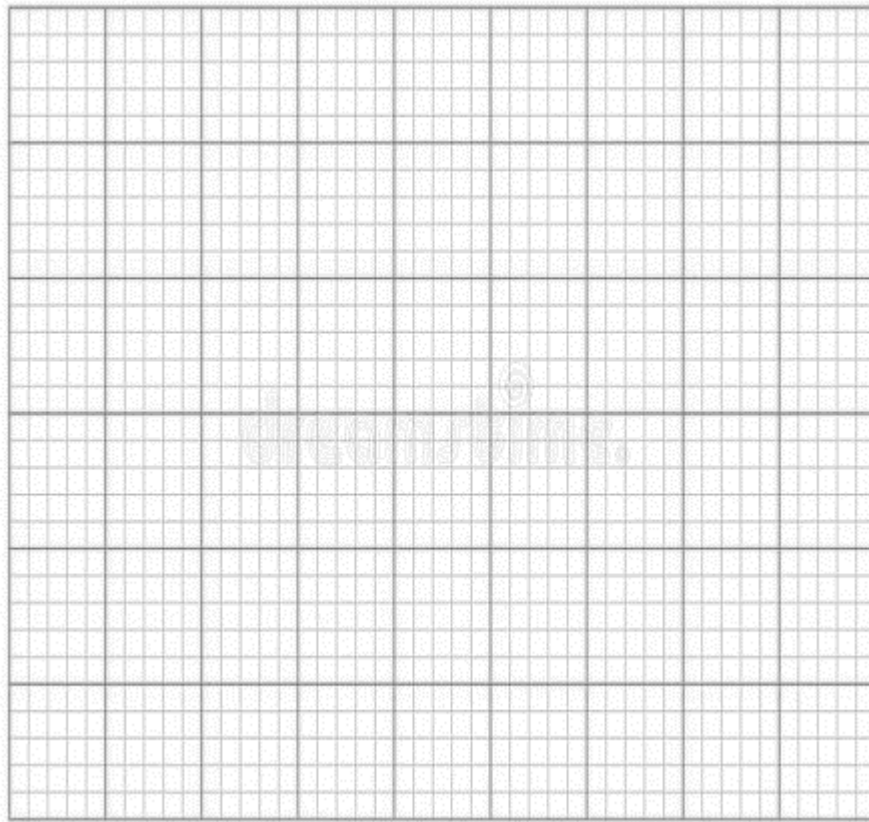
Students use a marble launcher to launch a marble at a speed of  $25 \text{ m s}^{-1}$  and try to demonstrate a relationship between angle and range and they come up with the following results:

Angle of elevation ( $^{\circ}$ )	Time of flight (s)	Horizontal component of velocity ( $\text{m s}^{-1}$ )	Range (m)
10	0.53		
20	1.05		
30	1.53		
40	1.97		
50	2.35		
60	2.65		
70	2.88		
80	3.01		

(a) Fill in the 2 blank columns with the expected values.

**Sample Problem 5.2.4 continues over the page**

(b) Construct a graph which illustrates the relationship between angle of elevation and range.



(c) Write a suitable conclusion for the experiment based on your answer to part (b).

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(d) According to your graph, what angle of elevation would produce maximum range for the projectile?

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(e) Prove that your answer to part (d) is correct.

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### Sample Problem 5.2.5 (Practical Investigation)

Students are asked to find a relationship between the range of a projectile and its horizontal launch velocity.

(a) Identify some variables which you should control in an experiment like this.

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(b) Identify your independent variable.

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(c) Identify your dependent variable.

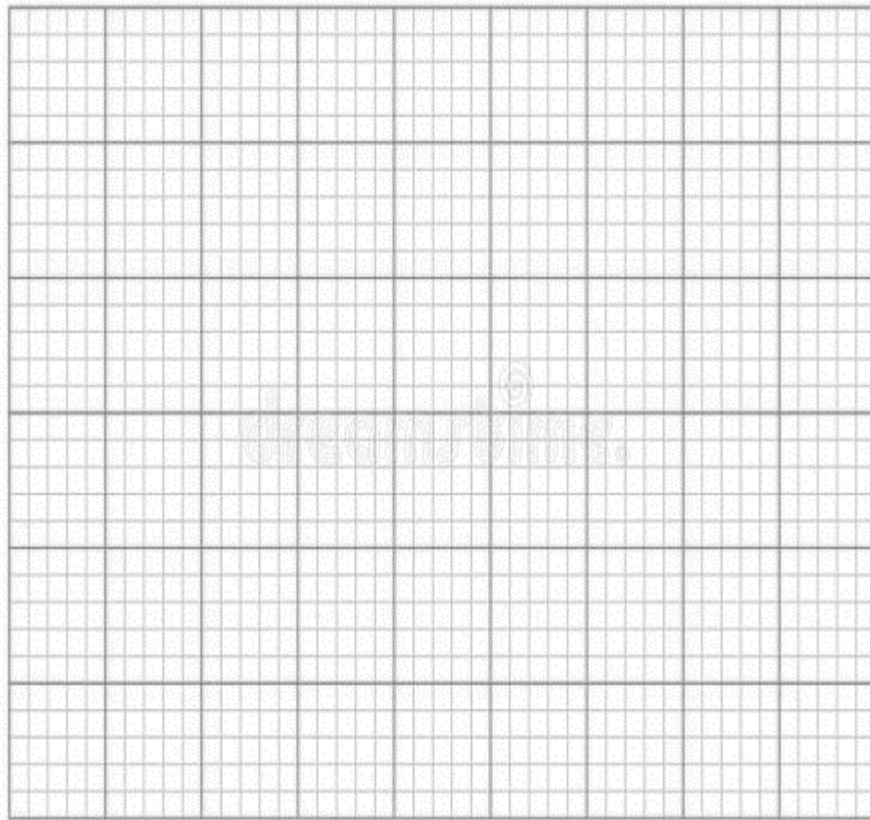
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The students decide to roll the ball off a desk at different speeds and measure the range using a ruler. The following results are obtained:

Launch speed ( $\text{m s}^{-1}$ )	Range (cm)
1.0	62
1.5	83
2.3	120
2.9	142
3.5	170
4.2	199
5.0	230

(d) Plot an appropriate graph to demonstrate the relationship which the students were investigating.



(e) Are any random errors evident in your results? Justify.

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(f) Are any systematic errors evident in your results? Justify.

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(g) Comment on the validity of the experiment.

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(h) Use the graph to determine the height of the table.

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# Consolidation / Extra Questions

1. An object is dropped from a stationary position at a height of 250m. Calculate:

a. The time it takes for the object to fall to the ground

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b. The velocity of the ball at impact

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2. A ball is propelled vertically upwards from ground level with an initial velocity of  $12.5\text{ms}^{-1}$ .

Calculate:

a. Velocity after 1.5 seconds

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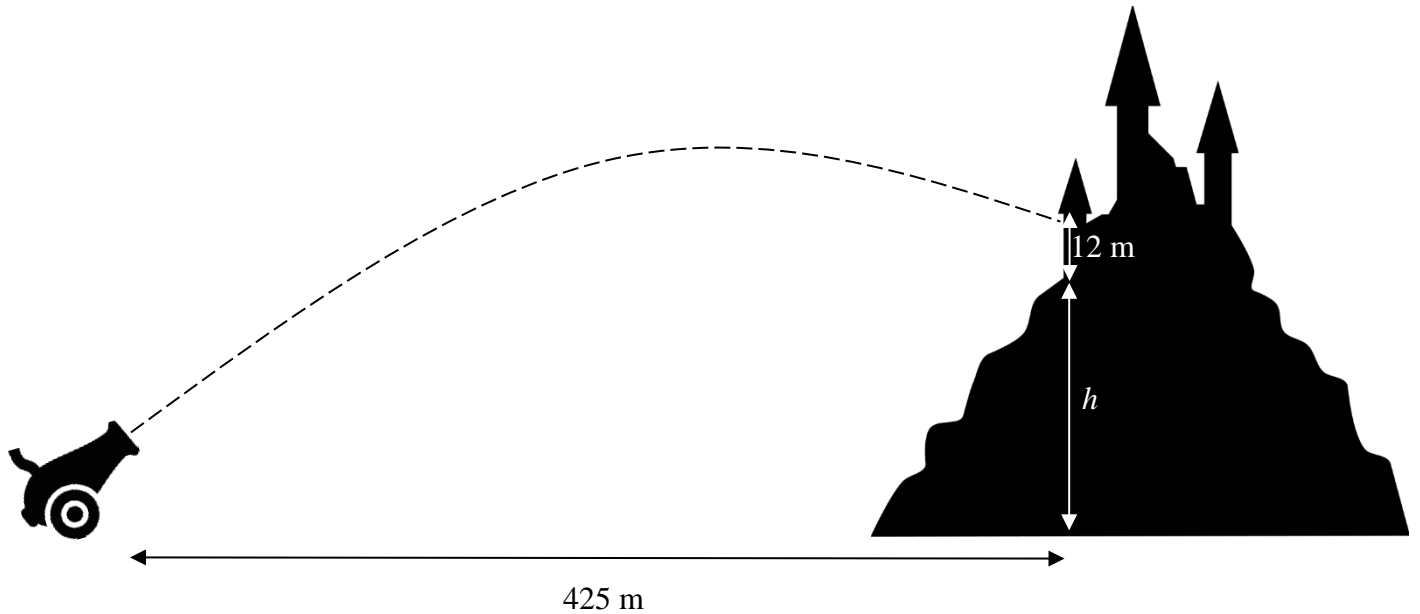
b. The maximum height reached

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c. The total time of flight

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8. A cannonball is fired at  $150 \text{ m s}^{-1}$  at an angle of  $55^\circ$  above the horizontal. It strikes the side of a castle at a height of  $12 \text{ m}$  above the surface. What is the height,  $h$ , of the hill that the castle is built on if the base of the castle is  $425 \text{ m}$  horizontally from the point where the cannonball is fired? Neglect the height of the cannon itself



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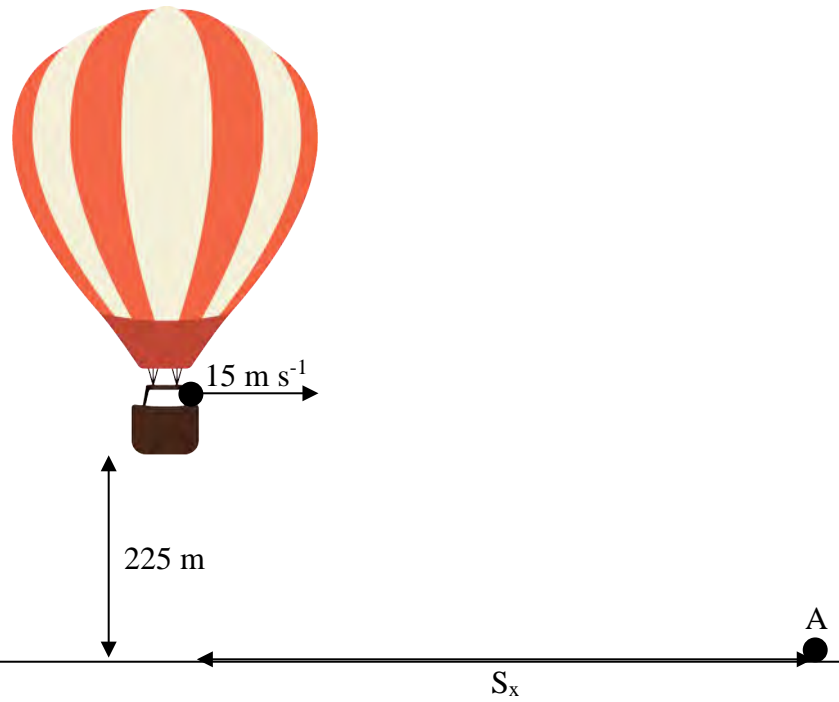
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9. A hot air balloon takes off from the ground with an acceleration of  $2\text{ms}^{-2}$ .  
When the hot air balloon is at an altitude of  $225\text{m}$ , an object is projected at  $15\text{ms}^{-1}$  horizontally according to the person on the hot air balloon and eventually strikes the ground at point A



- (a) Sketch the path of the ball as seen by an observer on the ground.
- (b) Find the maximum height above the ground which the ball reaches.

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- (c) Find the horizontal distance travelled by the ball.

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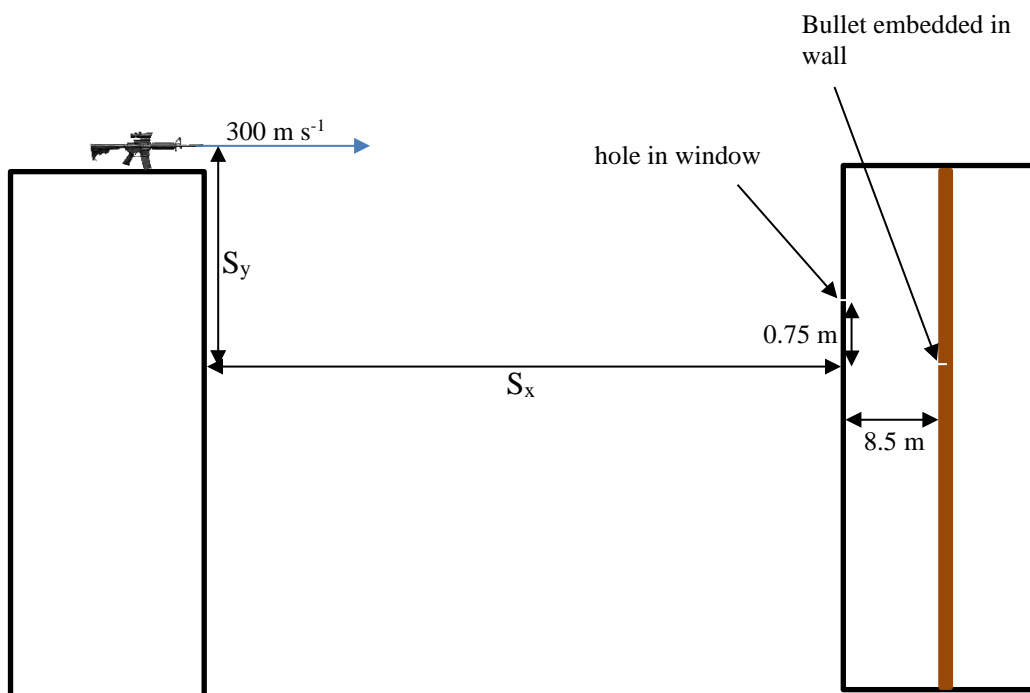
10. From the top of a tall building, a gun is fired.

The bullet leaves the gun at a speed of  $300 \text{ m s}^{-1}$ , parallel to the ground.

As the drawing shows, the bullet puts a hole in a window of another building and hits the wall that faces the window.

Using the data in the drawing, determine the distances  $S_x$  and  $S_y$ , which locate the point where the gun was fired.

Assume that the bullet does not slow down as it passes through the window.



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## Consolidation / Extra Questions

1.

(a) 7.14s

(b)  $70\text{ms}^{-1}$  down

2.

(a)  $2.2\text{ms}^{-1}$  down

(b) 8m

(c) 2.55s

3.

(a) 2874m

(b)  $177\text{ms}^{-1}$  @  $16^\circ$  below horizontal

4. 2.89m

5.

(a) 200m

(b) 56.6km

(c) 65.44km

(d)  $\theta = 0.6^\circ, 89.5^\circ$

# SCIENCE EDUCATION

HSC PHYSICS. SUMMED UP.